(2024 - Batch onwards)

**PS1IPHC600** Reg No : [ ]

St Aloysius (Deemed to be University)
Mangaluru
School of Physical Sciences (PG Programme)
Semester III - PG Examination - M.Sc. Mathematics
October / November - 2025
Complex Analysis I

Time : 2½ Hours Max. Marks : 60

## SECTION - A

**Answer any FIVE of the following.** (5x2=10)

1. Is the function $f(z) = \bar{z}$ analytic? Justify your answer.
2. Show that $||a| - |b|| \leq |a - b|$ where $a, b \in \mathbb{C}$.
3. Prove that an analytic function in a region whose derivative vanishes identically reduces to a constant.
4. Let $f$ be complex valued function defined on a domain $D$. Let $\lim_{x \to a} f(x) = A$, then prove that $\lim_{x \to a} im(f(x)) = im(A)$ where $im$ represents the imaginary part.
5. When do we say that the points $z$ and $z^*$ are symmetric with respect to the circle through $z_1, z_2, z_3$?
6. Evaluate $\int_{|z|=1} \frac{e^z}{z} dz$.



## SECTION - B

**Answer any FIVE of the following.** (5x10=50)

7. State and prove Cauchy's inequality. Also prove that $|a_1 + a_2 + \ldots + a_n| = |a_1| + |a_2| + \ldots + |a_n|$ if and only if $\frac{a_i}{a_j} > 0$, for each $i, j$, where $a_i \in \mathbb{C}, 1 \leq i \leq n$. **(6+4)**
8. a) Let $f(z) = u(z) + iv(z)$ be defined on an open subset $\Omega$ of $\mathbb{C}$ such that $u$ and $v$ have continous first order partial derivatives. If $u$ and $v$ satisfies the Cauchy Riemann equation, then prove that $f(z)$ is analytic on $\Omega$.
   b) State and prove Lucas theorem. **(5+5)**
9. Let $\{f_n\}$ be a sequence of continuous complex valued function defined on a subset $E$ of $\mathbb{C}$. Let $f$ be a complex valued function defined on a subset $E$ of $\mathbb{C}$ such that $f_n$ converges to $f$ uniformly on $E$, then prove that $f$ is continous on $E$. Also state and prove necessary and suffiecient condition for uniform convergence of a sequence of complex valued function defined on a subset $E$ of $\mathbb{C}$. **(5+5)**
10. Prove that derivative of an analytic fuction is also analytic.
11. State and prove the Cauchy's theorem for a rectangle.
12. State and prove the Cauchy's integral formula and compute $\int_{|z|=2} \frac{dz}{z^2 + 1}$.
13. Prove that any linear transformation maps circles to circles.
14. If $f(z)$ is continuous on a closed and bounded set $E$ and analytic in the interior of $E$, then show that the maximum of $|f(z)|$ on $E$ is assumed on the boundary of $E$. Also prove that a non constant analytic function maps open sets onto open sets. **(6+4)**

*******

(2024 - Batch onwards)

PS1IPHC601 Reg No : ☐☐☐☐☐☐☐☐

# St Aloysius (Deemed to be University)
# Mangaluru
## School of Physical Sciences (PG Programme)
## Semester III - PG Examination - M.Sc Mathematics
## October / November - 2025
## Topology

Time : 2½ Hours Max. Marks : 60

### SECTION - A

**Answer any FIVE of the following.** **(5x2=10)**

1. Let $X = \{a, b, c\}$. Check whether the following collection is a topology or not:
   a) $\tau_1 = \{\{a\}, \{c\}, \{a, b\}, \{a, c\}\}$
   b) $\tau_2 = \{\{a\}, \{b\}, X\}$
2. Let $A, B$ denote subsets of a space $X$. Prove that if $A \subseteq B$ then $\bar{A} \subseteq \bar{B}$.
3. Let $X$ and $Y$ be two topological spaces. Prove that the map $\pi : X \times Y \to X$ defined by $\pi(x, y) = x$ is continuous.
4. Let $X = \{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. Show that $X$ is compact.
5. Let $f : X \to X$ be a continuous map. Show that if $X = [0, 1]$, then there exists a point $x \in X$ such that $f(x) = x$.
6. Prove or disprove: A First countable space need not be second countable.



### SECTION - B

**Answer any FIVE of the following.** **(5x10=50)**

7. Let $X$ be a topological space. Suppose that $\mathscr{C}$ is a collection of open sets of $X$ such that for each open set $U$ of $X$ and each $x \in U$ there is an element $C \in \mathscr{C}$ such that $x \in C \subseteq U$. Prove that $\mathscr{C}$ is a basis for the topology of $X$. Also, if $\mathscr{B}$ and $\mathscr{B}'$ are bases for the topologies $\tau$ and $\tau'$ respectively on the set $X$, then show that the following are equivalent:
   i) $\tau'$ is finer than $\tau$.
   ii) For each $x \in X$ and each basis element $B \in \mathscr{B}$ containing $x$, there is a basis element $B' \in \mathscr{B}'$ such that $x \in B' \subseteq B$. **(6+4)**

8. Let $Y$ be a subspace of $X$. Then prove the following:
   a) The collection $\tau_y = \{Y \cap U / U \in \tau\}$ is a topology.
   b) A set is closed in $Y$ if and only if it equals the intersection of a closed set of $X$ with $Y$.
   c) If $\mathscr{B}$ is a basis for a topology $\tau$ on $X$ then the collection $\mathscr{B}_y = \{Y \cap B / B \in \mathscr{B}\}$ is a basis for subspace topology $\tau_y$ on $Y$. **(4+4+2)**
9. Let $X$ be a hausdorff space. Then prove the following:
   a) Each singleton set $\{x\}$ is closed in $X$.
   b) A sequence of points of $X$ converges to at most one point of $X$.
   c) A subspace of a Hausdorff space is again a Hausdorff space. **(3+3+4)**
10. State and prove the sequence lemma for a topological space.
11. Let $X$ be an ordered set having the least upper bound property in the order topology. Prove that each closed interval in $X$ is compact.
12. State and prove the uniform continuity theorem.
13. Prove the following:
    a) A space $X$ is connected if and only if the only subsets of $X$ that are both open and closed in $X$ are the empty set and $X$ itself.
    b) The union of a collection of connected subspaces of $X$ that have a point in common is connected. **(5+5)**
14. Prove the following:
    a) Every normal space is regular.
    b) Every metrizable space is normal. **(3+7)**

*******

(2024 - Batch onwards)

PS1IPHC602 Reg No : [ ]

# St Aloysius (Deemed to be University)
# Mangaluru
## School of Physical Sciences (PG Programme)
## Semester III - PG Examination - M.Sc. Mathematics
## October / November - 2025
## Numerical Analysis with Computational Lab

Time : 2½ Hours Max. Marks : 60

### SECTION - A

**Answer any FIVE of the following.** **(5x2=10)**

1. State Chebyshev's method to approximate a root of the equation $f(x) = 0$.
2. Define a simple root and a multiple root. Find the multiplicity of the root $x = 2$ in $x^3 - 3x^2 + 4 = 0$.
3. Find the linear Lagrange interpolating polynomial, given $f(2) = 4, f(2.5) = 5.5$ and hence find $f(2.2)$.
4. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using composite Trapezoidal rule with 4 sub intervals.
5. Using Runge–Kutta method of 2nd order, find an approximate value of $y(0.2)$ for the IVP $y' = x + y,\ y(0) = 1$ taking step size $h = 0.2$.
6. Solve the IVP $u' = -2tu^2,\ u(0) = 1$ using Euler's method. Find $u(0.2)$ using step size $h = 0.2$.



### SECTION - B

**Answer any FIVE of the following.** **(5x10=50)**

7. (a) If $\phi(x)$ is a contiuous function on some interval $[a_0, b_0]$ that contains the root and $|\phi'(x)| \leq \rho < 1$ in $[a_0, b_0]$ then for any choice of $x_0 \in [a_0, b_0]$ prove that $\{x_n\}$ determined from $x_{n+1} = \phi(x_n)$converges to the root $\alpha$ of $x = \phi(x)$.

   (b) Perform 3 iterations of the Bairstow method to extract the quadratic factor $x^2 + pq + q$ from the polynomial $p(x) = x^3 + x^2 - x + 2$. Use initial approximation $p_0 = -0.9$ and $q_0 = 0.9$ **(6+4)**
8. (a) Derive Secant method to find the root of the equation $f(x) = 0$ on the interval $[a, b]$.
   (b) Use Secant method to find the root of the equation $f(x) = x^3 - x - 1$, whose root lies in the interval $(0, 1)$. **(5+5)**
9. (a) State and prove Gerschgorin theorem.
   (b) Find the smallest eigen value in magnitude of the matrix
   $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ using inverse power method. **(4+6)**
10. (a)Derive Gregory Newton Forward Interpolating polynomial.
    (b)Derive the truncation error bounds formula given by $|E(x)| \leq \frac{Mh^2}{8}$. **(5+5)**
11. Evaluate the integral $I = \int_0^1 \frac{dx}{2x^2+2x+1}$ by using Lobatto 3-point formula and Radau 3-point formula. Compare with the exact solution.
12. (a)Deduce the one-point and two-point formula for Gauss-Hermite integration method. Also evaluate $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2+x+1}$ using Gauss Hermite 2-point formula.
    (b)Derive 2-point Radau integration formula. **(5+5)**
13. (a) Derive the formula for midpoint method and hence solve the IVP$y' = \frac{1}{x-y}, y(0) = 1$, $h = 0.5$, using midpoint method to find $y(1.5)$.

    (b) Solve the IVP, $u' = -2u^2t$, $u(0) = 2$ to find $u(0.2)$ using R-K method of 4th order. Take $h = 0.2$. **(6+4)**
14. Derive Adam's Moulton method to solve an Initial value problem $u' = f(t, u)$, $u(t_0) = \eta_0$.

*******

(2024 - Batch onwards)

PS1IPSC621a                                        Reg No :

# St Aloysius (Deemed to be University)
# Mangaluru
## School of Physical Sciences (PG Programme)
## Semester III - PG Examination - M.Sc. Mathematics
## October / November - 2025
## Commutative Algebra

**Time : 2½ Hours** **Max. Marks : 60**

### SECTION - A

**Answer any FIVE of the following.** **(5x2=10)**

1. Define the Jacobson radical of a ring. What is the Jacobson radical of $\mathbb{R}$?
2. Define unit element in a ring. If $u$ is a unit and $x$ is a nilpotent in a ring, then show that $u+x$ is a unit.
3. Let $M$ be an $A$-module which is isomorphic to a quotient of $A^n$, for some $n \in \mathbb{N}$ then prove that $M$ is finitely generated.
4. Let $I$ be an ideal of a ring $A$ and $S$ be multiplicatively closed in $A$. Then show that $S^{-1}A = S^{-1}I$ if and only if $I$ meets $S$.
5. If $N, P$ are submodules of an $A-$module $M$, and $S$ is a multiplicatively closed set in $A$ then prove that $S^{-1}(N \cap P) = S^{-1}N \cap S^{-1}P$.
6. If $A$ is a noetherian ring and $S$ is a multiplicatively closed set in $A$, show that $S^{-1}A$ is noetherian.



### SECTION - B

**Answer any FIVE of the following.** **(5x10=50)**

7. (a) Let $I$ and $J$ are ideals of $A$ which are co-prime then show that $I \cap J = IJ$.
   (b)Let $A$ be a ring and $I_1, I_2, \ldots, I_n$ be ideals of $A$. Prove that the map $\phi : A \to \prod_{j=1}^{n} A/I_j$ is surjective if and only if $I_j, I_k$ are co-prime for all $1 \leq j \neq k \leq n$. **(4+6)**
8. (a) Prove that every non-zero ring has atleast one maximal ideal.
   (b) Let $A$ be a ring and $M$ be a maximal ideal of $A$ such that every element of $1+M$ is a unit in $A$.Then show that $A$ is a local ring. **(6+4)**
9. (a) If $D$ denotes the set of all zero divisors of a ring $A$, then prove that $D = \bigcup_{x \neq 0} r(Ann(x))$.
   (b) Prove that for the ideals $I$ and $J$ of a ring $A$, $r(I+J) = r(r(I)+r(J))$.**(5+5)**
10. (a) Let $M, M'$, and $M''$ be $A$-modules. Prove that the sequence, $0 \to M' \xrightarrow{f} M \xrightarrow{g} M'' \to 0$, is exact if and only if $f$ is injective, $g$ is surjective and $g$ induces an isomorphism of $M/Imf$ onto $M''$.
    (b) Let $f : M \to N$ be an $A$-module homomorphism then show that $M/Kerf \cong Imgf$ **(5+5)**
11. (a) Let $f : A \to B$ be a ring homomorphism and let $\mathcal{P}$ be a prime ideal of $A$. Prove that $\mathcal{P}$ is a contraction of a prime ideal of $B$ if and only if $\mathcal{P}^{ec} = \mathcal{P}$.
    (b) If $I, J$ are ideals of a ring $A$, with $J$ finitely generated then show that $S^{-1}(I : J) = (S^{-1}I : S^{-1}J)$. **(6+4)**

contd...2/

12. (a) Prove that every ideal in $S^{-1}A$ is an extended ideal.
(b)Let $A$ be a ring and let $\phi : M \to N$ be an $A-$module homomorphism. For a prime ideal $\mathcal{P}$ let $M_{\mathcal{P}}$ denote the $S^{-1}A-$module $S^{-1}M$, where $S = A - \mathcal{P}$. Prove that the following statements are equivalent:

i) $\phi$ is injective

ii) $\phi_{\mathcal{P}} : M_{\mathcal{P}} \to N_{\mathcal{P}}$ is injective for each prime ideal $\mathcal{P}$ of $A$

iii) $\phi_{\mathcal{M}} : M_{\mathcal{M}} \to N_{\mathcal{M}}$ is injective for each maximal ideal M of $A$ .**(4+6)**

13. (a) For a prime ideal $\mathcal{P}$ of $A$, prove that the ring of fractions $A_{\mathcal{P}}$ is a local ring

(b) Let $g : A \to B$ be a ring homomorphism and $S$ be a multiplicatively closed set in $A$ such that $g(s)$ is a unit in $B$ for every $s \in S$. Prove that there is a unique ring homomorphism $h : S^{-1}A \to B$ such that $g = h \circ f$,where $f : A \to S^{-1}A$ given by $f(a) = \frac{a}{1}, a \in A$ .**(4+6)**

14. (a) Prove that in a Noetherian ring $A$ every ideal is a finite intersection of irreducible ideals.
(b) State and prove Second Uniqueness Theorem for primary decomposition.**(4+6)**

*******