

**St Aloysius (Deemed to be University)**  
**Mangaluru**  
**School of Physical Sciences (PG Programme)**  
**Semester I - PG Examination - M.Sc. Mathematics**  
**October / November - 2025**  
**Algebra I**

Time : 2½ Hours

Max. Marks : 60

**SECTION - A**Answer any **FIVE** of the following.**(5x2=10)**

1. Let  $G$  be a group and  $a, b$  be any two elements in the group  $G$ , then prove that  $(ab)^{-1} = b^{-1}a^{-1} \forall a, b \in G$ .
2. If  $\phi : G \rightarrow G'$  is a group homomorphism of groups, then group that  $|G| \leq |ker\phi| + |Im\phi|$ .
3. Let  $H$  and  $K$  be any two subgroups of a group  $G$ . Then show that  $[H : H \cap K] \leq [G : K]$ .
4. Let  $G$  be a group and  $S$  be the set of all sub groups of  $\bar{G}$ , then prove that  $G$  acts on  $S$ .
5. Prove that a  $p$ -group has a nontrivial center.
6. Define a characteristic of a ring. If  $R$  is a ring with unity then prove that  $\text{char}R > 0$  if and only if  $n.1 = 0$

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**SECTION - B**Answer any **FIVE** of the following.**(5x10=50)**

7. Prove the following :
  - a) A subgroup  $N$  of a group  $G$  is normal if and only if  $gNg^{-1} = N, \forall g \in G$ .
  - b) A subgroup  $N$  of a group  $G$  is normal if and only if  $gN = Ng, \forall g \in G$ .
  - c)  $N$  is a normal subgroup of  $G$  if and only if product of right cosets of  $N$  in  $G$  is again a right coset of  $N$  in  $G$ . **(3+3+4)**
8. State and prove the following:
  - a) Third isomorphism theorem.
  - b) Correspondence theorem for group. **(5+5)**
9. Define orbit of an element. Let  $G$  be a finite group,  $S$  be a non-empty set such that  $G$  acts on  $S$  and let  $s \in S$ . Then prove that the order of orbit of  $s$  is the index of the stabilizer of  $s$  in  $G$ , i.e.,  $|O_s| = [G : G_s]$ . Hence show that  $O(G) = O(O_s) \cdot O(G_s)$ . **(2+4+4)**
10. State and prove second and third sylow theorem. **(4+6)**
11. Prove that every group of order  $p^2$  is abelian where  $p$  is a prime number. Also prove that any group of order  $p^2$  is cyclic or is isomorphic to a product of two cyclic groups each of order  $p$ . **(4+6)**
12. Prove that every group of order 15 and 35 are cyclic. **(5+5)**
13. If  $R$  is a commutative ring with identity and  $I$  is an ideal of  $R$ , then prove that  $R/I = \{x + I : x \in R\}$  is a commutative ring with identity. Also Prove that for any commutative ring  $R$  with identity, an ideal  $P$  of  $R$  is a prime ideal of  $R$  if and only if  $R/P$  is an integral domain. **(5+5)**
14. Prove the following:
  - a)  $\mathbb{Z}_p$  is a field, where  $p$  is a prime number.
  - b) Every finite integral domain is a field.
  - c) If  $R$  is a nonzero integral domain, then characteristic of  $R$  is either zero or prime number. **(3+3+4)**

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**Semester I - PG Examination - M.Sc. Mathematics**  
**October / November - 2025**  
**Linear Algebra I**

Time : 2½ Hours

Max. Marks : 60

**SECTION - A**Answer any **FIVE** of the following.**(5x2=10)**

1. For any  $n \times n$  matrix  $A$  prove that  $\det(A^t) = \det(A)$
2. Define elementary matrices and different types of elementary matrices.
3. Define a basis for a vector space. Also prove that  $\{(1, 2), (3, 4)\}$  forms a basis of  $\mathbb{R}^2$  over  $\mathbb{R}$ .
4. Find the dimension of the vector space of all  $n \times n$  matrices over a field  $F$  with trace zero.

5. Find the eigenvalues of  $\begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$  over the field  $\mathbb{R}$ .

6. Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear map given by  $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y \\ y + z \end{bmatrix}$ . Find Kernel of  $T$ . Is  $T$  a bijection? Justify

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**SECTION - B**Answer any **FIVE** of the following.**(5x10=50)**

7. (a) Write the properties of determinant function  
 (b) Let  $A, B$  be any two  $n \times n$  matrices, then prove that  $\det(AB) = \det(A) \det(B)$  **(4+6)**
8. (a) Prove that the system  $A^1 X = B^1$  and  $AX = B$  have same solutions, where  $[A^1|B^1]$  is obtained from  $[A|B]$  by applying finitely many elementary row operations.  
 (b) Show that the elementary matrices are invertible and their inverses are also elementary. **(4+6)**
9. (a) Let  $V$  be a vector space over a field  $F$  and let  $W_1, W_2$  be subspaces of  $V$ . Then prove that  $W_1, W_2$  are independent if and only if  $W_1 \cap W_2 = \{0\}$ .  
 (b) Let  $V$  be a finite dimensional vector space over a field  $F$ . If  $W$  is a subspace of  $V$  then prove that there exists a subspace  $W^1$  of  $V$  such that  $V = W \oplus W^1$ . **(4+6)**
10. (a) Let  $W$  be a subspace of a finite dimensional vector space  $V$ . Prove that  $W$  is finite dimensional and  $\dim W \leq \dim V$ , moreover  $\dim W = \dim V$  if and only if  $W = V$ .  
 (b) Find a basis for the subspace of  $\mathbb{R}^4$  spanned by the vectors  $(1, 2, -1, 0), (4, 8, -4, -3), (0, 1, 3, 4)$  and  $(2, 5, 1, 4)$ . **(6+4)**
11. (a) Let  $V$  be a finite-dimensional vector space over a field  $F$ . Prove that
  1. a finite set  $S$  which spans  $V$  contains a basis for  $V$
  2. a finite linearly independent set  $L$  can be extended to a basis of  $V$ .
 (b) Let  $V$  be a finite dimensional vector space over a field  $F$ . If  $S$  is a finite set that spans  $V$  and  $L$  is a finite linearly independent set in  $V$  then prove that  $|L| \leq |S|$ . **(5+5)**

Contd...2

12. (a) Let  $T : V \rightarrow W$  be a linear map of the vector spaces  $V, W$  over a field  $F$  of dimensions  $n, m$  respectively. Let  $A$  be the matrix of  $T$  with respect to the bases  $\mathcal{B}$  and  $\mathcal{C}$  of  $V$  and  $W$  respectively. Prove that the matrix of  $T$  with respect to any two bases of  $V$  and  $W$  is given by  $QAP^{-1}$  where  $Q \in GL_m(F)$  and  $P \in GL_n(F)$
- (b) Prove that the row rank and column rank of an  $m \times n$  matrix are equal. **(7+3)**
13. (a) Show that the eigenvectors corresponding to distinct eigenvalues are linearly independent.
- (b) Diagonalize the matrix  $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ . **(5+5)**
14. Prove the following are equivalent for a map  $m : \mathbb{R}^n \rightarrow \mathbb{R}^n$  :
1.  $m$  is a rigid motion of  $\mathbb{R}^n$  which fixes the origin.
  2.  $m$  preserves dot product.
  3.  $m$  is the left multiplication by an orthogonal matrix. **(10)**

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**Semester I - PG Examination - M.Sc. Mathematics**  
**October / November - 2025**  
**Operations Research**

Time : 2½ Hours

Max. Marks : 60

**SECTION - A**Answer any **FIVE** of the following.

(5x2=10)

1. Write the dual of the following:

$$\begin{aligned} &\text{Maximize } z_2 = x_1 - x_2 + 3x_3 \\ &\text{subject to } \begin{aligned} x_1 + x_2 + x_3 &\leq 10 \\ 2x_1 - x_2 - x_3 &\leq 2 \\ 2x_1 - 2x_2 - 3x_3 &\leq 6 \\ x_1, x_2, x_3 &\geq 0 \end{aligned} \end{aligned}$$

2. A pineapple firm produces two products-canned pineapple and canned juice. The specific amounts of material, labour and equipment required to produce each product and the availability of each of those resources are shown in the table given below:

	Canned Juice	Canned Fruit	Available resources
Labour(Hour)	3	2	12
Equipment(Hour)	1	2.3	6.9
Material (Unit)	1	1.4	4.9

Assuming one unit each of canned pineapple and canned juice has a profit margin of ₹ 1 and ₹ 2 respectively, form a linear programming problem to determine the units of the products to be produced to maximise the profit.

3. Obtain the initial basic feasible solution to the following transportation problem using the least cost method.

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_1$	1	2	3	4	6
$O_2$	4	3	2	5	8
$O_3$	5	2	2	1	10
Demand	4	6	8	6	

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4. Find an initial basic feasible solution of the following problem using the north west corner rule.

	Banglore	Nasik	Bhopal	Delhi	Capacity
Chennai	6	8	8	5	30
Madurai	5	11	9	7	40
Trichy	8	9	7	13	50
Demand	35	28	32	25	

5. Solve the payoff matrix :

Player A/Player B	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
$A_1$	-2	0	0	5	3
$A_2$	3	2	1	2	2
$A_3$	-4	-3	0	-2	6
$A_4$	5	3	-4	2	-6

6. Solve the following payoff matrix using the method of odds.

	$B_1$	$B_2$
$A_1$	1	5
$A_2$	4	2

SECTION - B

Answer any FIVE of the following.

(5x10=50)

7. Solve using the simplex method:  
 Minimize  $z = x_1 - 3x_2 + 2x_3$   
 subject to  $3x_1 - x_2 + 2x_3 \leq 7$   
 $-2x_1 + 4x_2 \leq 12$   
 $-4x_1 + 3x_2 + 8x_3 \leq 10$   
 $x_1, x_2, x_3 \geq 0$

8. Solve the following linear programming problems graphically:

- a) Maximize  $z = 8x_1 + 7x_2$   
 subject to  $3x_1 + x_2 \leq 66000$   
 $x_1 + x_2 \leq 45000$   
 $x_1 \leq 20000$   
 $x_2 \leq 40000$   
 $x_1, x_2 \geq 0$

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- b) Maximize  $z = 4x + 5y$   
 subject to  $x + y \leq 20$   
 $3x + 4y \leq 72$   
 $x, y \geq 0$

(5+5)

9. Solve using the Big M Method:

- Minimize  $z = -3x_1 + 2x_2$   
 subject to  $x_1 - 4x_2 \leq -14$   
 $-3x_1 + 2x_2 \leq 6$   
 $x_1, x_2 \geq 0$

10. Find the optimal transportation plan:

	I	II	III	IV	V	Available
A	7	6	4	5	9	40
B	8	5	6	7	8	30
C	6	8	9	6	5	20
D	5	7	7	8	6	20
Required	30	30	15	20	5	

11. Four different jobs can be done on four different machines. The matrix below gives the cost in rupees of producing a job  $i$  on the machine  $j$ . How should the jobs be assigned to the various machines so that the total cost is minimised? Also, formulate the mathematical model for the problem. Show all the working and steps.

	$M_1$	$M_2$	$M_3$	$M_4$
$J_1$	5	7	11	6
$J_2$	8	5	9	6
$J_3$	4	7	10	7
$J_4$	10	4	8	3

12. A product is produced by 4 factories A, B, C and D. The unit production cost in them are ₹ 2, ₹ 3, ₹ 1 and ₹ 5 respectively. Their production capacities are :  
 Factory A – 50 units, Factory B – 70 units, Factory C – 30 units and Factory D – 50 units. These factories supply the product to 4 stores, the demands of which are 25, 35, 105 and 20 units respectively. Unit transport cost in rupees from each factory to each store is given in the table below:

Factories/Stores	$S_1$	$S_2$	$S_3$	$S_4$
$F_1$	2	4	6	11
$F_2$	10	8	7	5
$F_3$	13	3	9	12
$F_4$	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores, so that the total production and transportation cost is minimum.

13. Explain the method of solving games by the rule of dominance and hence solve:

Player A/Player B	1	2	3	4	5
I	1	3	2	7	4
II	3	4	1	5	6
III	6	5	7	6	5
IV	2	0	6	3	1

14. Write down the algorithm for solving  $2 \times n$  matrix games using the graphical method.

Hence solve :

	$B_1$	$B_2$	$B_3$
$A_1$	1	3	12
$A_2$	8	6	2

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**Graph Theory**

Time : 2½ Hours

Max. Marks : 60

**SECTION - A**Answer any **FIVE** of the following.**(5x2=10)**

- Show that the sum of degrees of all points of a graph  $G$  is twice the number of lines in  $G$ .
- Define the complement  $\overline{G}$  of a connected graph  $G$ . Prove or disprove: If  $G_1$  and  $G_2$  are graphs then  $\overline{G_1 + G_2} = \overline{G_1} + \overline{G_2}$ .
- Prove that every nontrivial tree has atleast two end points.
- Define connectivity and line connectivity for a graph  $G$ .
- For any graph  $G$ , prove that  $\chi(G) \leq 1 + \Delta(G)$ , where  $\Delta(G)$  is the maximum degree of a point in  $G$ .
- If  $G$  is  $(p, q)$  plane graph with  $k$  components, then prove that  $p - q + r = k + 1$ .

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**SECTION - B**Answer any **FIVE** of the following.**(5x10=50)**

- (a) Let  $G$  be a graph. Prove that any closed walk of odd length in  $G$  contains an odd cycle.  
(b) For any graph  $G$  with six points prove that either  $G$  or  $\overline{G}$  contains a triangle. **(5+5)**
- Let  $G$  be a graph with  $p \geq 4$  points. Then prove that the intersection number  $\omega(G) = q$  if and only if  $G$  has no triangles.
- Let  $x$  be a line of a connected graph  $G$ . Prove that the following statements are equivalent :
  - $x$  is a bridge of  $G$ .
  - $x$  is not on any cycle of  $G$ .
  - There exists points  $u$  and  $v$  of  $G$  such that the line  $x$  is on every path joining  $u$  and  $v$ .
  - There exists a partition of  $V$  onto subsets  $U$  and  $W$  such that for any points  $u \in U$  and  $w \in W$ , the line  $x$  is on every path joining  $u$  and  $w$ .
- Let  $G$  be a connected graph with at least three points. Prove that  $G$  is a block if every two points of  $G$  lie on a common cycle. Show that a cubic graph  $G$  has a cut point if and only if it has a bridge. **(4+6)**
- Prove that the following statements are equivalent for a connected graph  $G$  :
  - $G$  is Eulerian.
  - every point of  $G$  has even degree.
  - the set of lines of  $G$  can be partitioned into cycles.
- Prove that in a connected graph the minimum number of points separating two non-adjacent points  $s$  and  $t$  is the maximum number of disjoint  $s - t$  paths.
- If  $G$  is a plane map with  $p$  vertices  $q$  edges and  $r$  faces, then prove that  $p - q + r = 2$ . Also prove that the graph  $K_5$  and  $K_{3,3}$  are nonplanar. **(5+5)**
- Prove that any planar graph is 5-colorable. Also for any graph  $G$  with  $p$  points, prove that
  - $2\sqrt{p} \leq \chi + \bar{\chi} \leq p + 1$
  - $p \leq \chi\bar{\chi} \leq \left(\frac{p+1}{2}\right)^2$

**(5+5)**

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