PS1IPHC550

Reg No :

St Aloysius (Deemed to be University)

Mangaluru

MSc Mathematics -SEMESTER II -PG EXAMINATION

APRIL - 2025

Algebra II

Time : 2½ Hours **Max. Marks : 60**

Answer any FIVE FULL questions **(12x5=60)**

1. a. Prove that every principal ideal domain(PID) is a unique factorization domain (UFD). Give an example of UFD, which is not a PID. 7

   b. Prove that every Euclidean domain is a PID. 5

2. a. Prove that in an integral domain the following are equivalent. 7
   i) Every non zero nonunit in R is a product of finitely many irreducible elements in R. 5
   ii) R does not contain an infinite strictly increasing chain of principal ideals.

   b. Let R be an integral domain. Prove or disprove the following:
   i) Every prime element in R is irreducible.
   ii) Every irreducible element in R is prime.

3. a. Prove that the polynomial ring $\mathbb{Z}[x]$ is a unique factorization domain. 8

   b. Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$ be an integer polynomial and p be a prime integer such that $p \nmid a_n$. If the residue $\bar{f}$ of f modulo p is an irreducible element in $F_p[x]$, then prove that f is irreducible element in $\mathbb{Q}[x]$. 4

4. a. Let L and K be extensions of a field F. Let $\alpha \in L$ and $\beta \in K$ be algebraic over F. Prove that there exists an F-isomorphism from $F(\alpha)$ to $F(\beta)$ which sends α to β if and only if α and β are the roots of the same irreducible polynomial over F. 8

   b. If α is algebraic over F having $f(x)$ as its minimal polynomial, then prove that $[F(\alpha) : F] = degf(x)$. 4

5. a. Prove that (a, b) in $\mathbb{R}^2$ is constructible if and only if a and b are constructible real numbers. 4

   b. If $F \subseteq K \subseteq L$ are field, then prove that $[L : F] = [L : K][K : F]$. Also determine $[\mathbb{Q}(3\sqrt{2}, 4\sqrt{5}) : \mathbb{Q}]$. 8

6. a. Let p be a prime number and n be any positive integer then prove that there exists a field with p^n elements. 6

   b. State and prove the fundamental theorem of algebra. 6

7. a. Prove that the set of all consructible real numbers forms a subfield of $\mathbb{R}$ containing $\mathbb{Q}$. 6

   b. Show that any two finite fields of the same order are isomorphic. 6

8. a. If K is a Galois extension of a field F, then prove that the fixed field of $G(K/F)$ is F. 6

   b. Let G be a finite group of automorphisms of a field K of characteristic zero and F be the fixed field of G. Then show that $[K : F] = O(G)$. 6

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Mangaluru
Semester II - P.G. Examination - M.Sc. Mathematics
April - 2025
REAL ANALYSIS II

Time : 2 ½ Hours Max. Marks : 60

Answer any FIVE Full questions from the following. (12x5=60)

1. a. Suppose f is bounded on $[a,b]$, f has finitely many points of discontinuity on $[a,b]$ and α is continuous at every point at which f is continuous. Then prove that $f \in \mathscr{R}(\alpha)$. (7)

   b. If $f \in \mathscr{R}(\alpha)$ on $[a,b]$ and if $|f(x)| \leq M, \forall x \in [a,b]$, then prove that $|\int_a^b f d\alpha| \leq M(\alpha(b) - \alpha(a))$. (5)

2. a. Suppose $f \in \mathscr{R}(\alpha)$ on $[a,b]$ and $m \leq f(x) \leq M, x \in [a,b]$. Suppose ϕ is continuous on $[m,M]$ and $h(x) = \phi(f(x))$ on $[a,b]$, then prove that $h \in \mathscr{R}(\alpha)$ on $[a,b]$. (7)

   b. Let f be a bounded function and let α be a monotonically increasing function on $[a,b]$. Then prove that $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha$. (5)

3. Prove that there exists a real continuous function on the real line which is nowhere differentiable. (12)

4. a. If K is a compact metric space, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, \ldots$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K. (5)

   b. Define a pointwise bounded sequence. If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a convergent subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$. (7)

5. a. Let (X,d) be a metric space and E be a subset of X. Suppose $f_n \to f$ uniformly on E. Let x be the limit point of E and suppose that $lim_{t \to x} f_n(t) = A_n, n \geq 1$, then prove that $\{A_n\}$ converges and $lim_{t \to x} f(t) = lim_{n \to \infty} A_n$. (7)

   b. Suppose A is an algebra of functions on a set E, A seperates points on E and A vanishes at no point of E. Suppose x_1, x_2 are distinct points of E and c_1, c_2 are constants. Then prove that A contains a function f such that $f(x_1) = c_1$ and $f(x_2) = c_2$. (5)

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6.
a. Show that $\int_0^\infty \frac{sinx}{x} dx$ is convergent. (4)
b. Examine the convergence of $\int_0^2 \frac{dx}{2x-x^2}$. (3)
c. State and prove Abel's Test. (5)

7.
a. If f and g are two functions on $[a,b]$ such that $lim_{x\to a^+} \frac{f(x)}{g(x)} = l$, where l is a non-zero finite number, then prove that the two integrals $\int_a^b f dx$ and $\int_a^b g dx$ converge or diverge together at a. (4)

b. Prove that the integral $\int_a^\infty f dx$ converges if and only if for every $\epsilon > 0$, there corresponds a positive number x_0 such that $|\int_{x_1}^{x_2} f dx| < \epsilon$, for all $x_1, x_2 \geq x_0$. (3)

c. If f and g are positive in $[a,x]$ and $lim_{x\to\infty} \frac{f(x)}{g(x)} = l$, where l is a non-zero finite number then show that the two integrals $\int_a^\infty f dx$ and $\int_a^\infty g dx$ converge or diverge together. (5)

8. State and prove the Implicit Function theorem with an example. (12)

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Semester II- P.G. Examination- M.Sc. Mathematics

April - 2025

RESEARCH METHODOLOGY AND ETHICS

Time: 2½ Hours Max Marks: 60

Answer FIVE FULL questions. (12x5=60)

1. a. Write a short note on:
   i) Objectives of research
   ii) motivation in research (6)

   b. What do you mean by research? Explain its significance in modern times. (6)

2. a. Explain the benefits of literature review in research. (6)

   b. Distinguish between research methods and research methodology. (6)

3. a. Describe the structure of a research report in detail. (8)

   b. Outline the key elements of a bibliography and explain its role in academic research. Discuss the difference between a bibliography and a reference list, including their formatting and content requirements. (4)

4. a. Explain the principle of mathematical induction and how it is used to prove mathematical statements and theorems involving integers or other discrete structures. Provide examples of mathematical statements that can be proved using mathematical induction. (6)

   b. Discuss the significance of clarity, precision, and logical coherence in mathematical writing. (6)

5. a. Determine whether these system specifications are consistent: "Whenever the system software is being upgraded, users cannot access the file system. If users access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded". (6)

   b. Construct the truth table for each of the following compound propositions.
   i) $(p \to q) \to (\neg q \to \neg p)$
   ii) $(p \lor q) \to (p \land q)$ (6)

6. a. Describe the concept of Intellectual Property Rights. Explain its origin and importance. (8)

   b. Explain the role and importance of research databases in academic research. Discuss the various types of research databases available. (4)

7. a. What does scientific misconduct refer to? Discuss the various forms of scientific misconduct. (8)

   b. What are research ethics? Explain the need for ethics in academic research. (4)

8. a. Describe the roles of co-authors, contributors, and acknowledgments in scholarly publications. Provide examples of ethical dilemmas related to authorship and contributorship. (6)

   b. Explain the concept of copyright and its importance in academic and creative works. Discuss the rights and responsibilities of creators and users of copyrighted materials. (6)

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Mangaluru

Semester II - P.G. Examination - M.Sc. Mathematics

April - 2025

ORDINARY DIFFERENTIAL EQUATIONS

Time : 2 ½Hours

Max. Marks : 60

(12x5=60)

Answer FIVE FULL questions

1. a. Verify that $\sin(x), \sin(2x)$, and $\sin(3x)$ are linearly independent on the real line. (4)

   b. Using the method of undetermined coefficients, find a particular solution of $x^{(4)} + 4x = 2\sin(t) + 1 + 3t^2 + 4e^t$. Also give its general solution. (8)

2. a. Let φ_1 be a solution of $L_2(x) = a_0(t)x'' + a_1(t)x' + a_2(t)x = 0, t \in I$ where $a_0(t) \neq 0, \forall\ t \in I$ and $\varphi_1(t) \neq 0, \forall\ t \in I$. Then show that
$$\varphi_2(t) = \varphi_1(t)\int_{t_0}^{t_1} \frac{1}{(\varphi_1(s))^2} exp\left[-\int_{t_0}^{s} \frac{a_1(\xi)}{a_0(\xi)}\, d\xi\right] ds$$
$,t_0 \in I$, is a solution of the above equation on I and $t_0 \in I$. Further show that $\varphi_1(t)$ and $\varphi_2(t)$ are linearly independent on I. (8)

   b. Consider the equation $L_2(x) = x'' - 2tx' + 2x = 0$. Given that $\phi_1(t) = t$ is a solution. Find the second linearly independent solution. (4)

3. a. Find a particular solution of $L_3(x) = x''' - 3x' + 4x' - 2x = e^t \sec(t), 0 \leq t \leq \frac{\pi}{2}$. Given that $e^t, e^t\cos(t), e^t\sin(t)$ are linearly independent solution of $L_3(x) = 0$. (8)

   b. Prove that if the wronskian of n functions $x_1(t), x_2(t), \ldots . x_n(t)$ defined on an interval $I \subseteq \Re$ is non zero for atleast one point of I, then the above n functions are linearly independent on I. Verify whether the converse is true. (4)

4. Prove that the Legendre polynomial $P_n(x)$ are given by $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. Also find the Legendre series expansion of $f(x) = e^x\ on\ [-1, 1]$. (12)

5. a. Prove the following property of Bessel function. $\frac{d}{dt}(t^p J_p(t)) = t^p J_{p-1}(t)$. Hence express $J_5(t)$ in terms of $J_0(t)$ and $J_1(t)$. (8)

   b. Find the nature of the singular points on the real line for the equation $(t-1)^2 t(t-3)x'' + tx' + t^2 x = 0$. (4)

6. a. Find e^{tA} when $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$. (6)

   b. Let Φ be a fundamental matrix of the system $X'(t) = A(t)X(t)\ \forall t \in I$, Then prove that particular solution $\psi(t)$ of the non homogenous system $X'(t) = A(t)X(t) + b(t), t \in I$ is given by $\psi(t) = \phi(t)\int_{t_0}^{t} \Phi^{-1}(s)b(s)ds$, such that for some $t_0 \in I, \psi(t_0) = 0$. (6)

Contd...2

7. Show that $\Phi(t) = \begin{bmatrix} e^{3t} & 2te^{3t} \\ 0 & e^{3t} \end{bmatrix}$ is a fundamental matrix of the system $X'(t) = A(t)X$, where $A(t) = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$ and $B(t) = \begin{bmatrix} e^{t} \\ e^{-t} \end{bmatrix}$. (12)

Further find the solution of $X'(t) = A(t)X(t) + B(t)$ satisfying the condition $X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

8. a. Let $f(t,x)$ be a continuous function defined over a rectangle $R = \{(t,x) : |t - t_0| \le p, |x - x_0| \le q\}$, where p, q are some positive real numbers. Let $\frac{\partial f}{\partial x}$ be defined, continuous and bounded on R, then prove that $f(t,x)$ satisfies Lipschitz condition on R. (4)

b. Define Lipschitz condition and show that $f(t,x) = x^{1/2}$ satisfies the Lipschitz condition on $R = \{(t,x) : |t| \le 2, |x| \le 2\}$. (4)

c. state and prove Grownwall's inequality. (4)

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