PS1IPHC500 Reg No :

St Aloysius (Deemed to be University)
Mangaluru
Semester I – P.G. Examination – M.Sc. Mathematics
November - 2024
Algebra I

Time : 2 ½ Hours **Max. Marks : 60**

Answer FIVE FULL questions (12x5=60)

1. a. Let C_n be a cyclic group of order n and $n = rs$ with $\gcd(r, s) = 1$. Then prove that $C_n \cong C_r \times C_s$, where C_r and C_s are cyclic groups of order r and s respectively. 7

   b. Define a normal subgroup of a group. Prove the following : 5

      1. A subgroup N of a group G is normal if and only if $gNg^{-1} = N, \forall g \in G$.

      2. A subgroup N of a group G is normal if and only if $gN = Ng, \forall g \in G$.

2. a. Define kernel of a homomorphism. If $\phi : G \to G'$ is a homomorphism then prove that $\ker \phi$ is a normal subgroup of G . 6

   b. Let H and K be any two finite subgroups of a group G . Then prove that $O(HK) = \frac{O(H) \,.\, O(K)}{O(H \cap K)}$. 6

3. a. State and prove third isomorphism theorem. 6

   b. For any integer b , prove that the subset $b\mathbb{Z}$ is a subgroup of $(\mathbb{Z}, +)$. Moreover show that any subgroup of $(\mathbb{Z}, +)$ is of the type $b\mathbb{Z}$ for some integer b . 6

4. a. State and prove Cayley's theorem. 7

   b. Let H be a subgroup of a group G and let S denote the set of all left cosets of H in G. Then prove that there exists a homomorphism $\phi : G \to Perm(S)$ such that $\ker \phi \subseteq H$. 5

5. a. Show that any group of order $13^2 \times 17^2$ is abelian. 4

   b. Prove that any group of order p^2 is abelian where p is a prime number. 4

   c. Prove that any group of order p^2 is cyclic or isomorphic to product of groups each of order p. 4

6. a. Write all possible class equations of a group of order 21. 5

   b. State and prove first Sylow theorem. 7

7. a. Prove that every integral domain can be embedded in a field. 9

   b. Prove that every finite integral domain is a field. 3

8. a. If R is a commutative ring with identity and I is an ideal of R, then prove that $R/I = \{x + I : x \in R\}$ is a commutative ring with identity. 5

   b. Prove the following: 7

      1. The set of all units of a ring R with identity is a group with respect to multiplication in R .

      2. If R is a non-zero integral domain then characteristic of R is either zero or a prime number.

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St Aloysius (Deemed to be University)

Mangaluru

M.Sc. Mathematics - SEMESTER I PG EXAMINATION

November - 2024

Linear Algebra I

Time : 2 ½ Hours Max. Marks : 60

Answer FIVE FULL questions (12x5=60)

1. a. For any $n \times n$ matrix A, prove that $A(\text{adj}A) = (\text{adj}A)A = \det(A)I_n$. 9

   b. Define a permutation matrix. Show that a permutation matrix is invertible. 3

2. a. Prove that $\det(AB) = \det A \det B$ for any two $n \times n$ matrices A and B. 7

   b. Prove that a square row echelon matrix is either the identity matrix or else its bottom row is zero. Hence derive that a square matrix is invertible if and only if its row echelon form is the identity matrix. 5

3. a. Prove that for any $n \times n$ matrix A, $\det(A^t) = \det(A)$. 5

   b. Let A be an $n \times n$ matrix with integer entries. Prove that A is invertible, and that A^{-1} has integer entries if and only if $\det A = \pm 1$. 4

   c. Write the permutation matrix associated with the permutation $p = (1\ 4\ 3) \in S_5$. Write p as a product of transpositions and and determine sign p. 3

4. a. Let V be a finite-dimensional vector space over a field F. Prove that 7
   1. a finite set S which spans Vcontains a basis for V
   2. a finite linearly independent L set an be extended to a basis of V.

   b. If W is a subspace of a finite-dimensional vector space Vover a field F then prove that $\dim W \leq \dim V$. 5

5. a. Let F be a field. Prove that the columns of a $n \times n$ matrix A forms a basis for F^n over F if and only if A is invertible. 4

   b. If W_1 and W_2 are subspaces of a finite dimensional vector space V over a field F, then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$ 8

6. a. Let $W \subseteq \mathbb{R}^4$ be the solution space of the equation $AX = 0$, where $A = \begin{bmatrix} 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$. Find a basis for W. 5

   b. Let V be an $n-$dimensional vector space and let $\mathcal{B}$ be an ordered basis of V. Prove that the collection of all ordered bases of V is $\{\mathcal{B}P : P \in GL_n(F)\}$. 7

7. a. State and prove the rank-nullity theorm. 6

   b. Let $T : V \to W$ be a linear transformation between finite dimensional vector spaces. Prove that there exist bases $\mathcal{B}$ and $\mathcal{C}$ of V and W respectively such that the matrix of T with respect to these basis is of the form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, where I_r is the $r \times r$ identity matrix and r is the rank of T. 6

8. a. Show that the following are equivalent for $A \in M_n(\mathbb{R})$: 7
   i) $A \in O_n(\mathbb{R})$
   ii) $AX \cdot AY = X \cdot Y$ for all $X, Y \in \mathbb{R}^n$.
   iii) Columns of A are mutually orthogonal unit vectors in $\mathbb{R}^n$.

   b. Compute the characteristic polynomial, eigenvalues, and eigenvectors of the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$. 5

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PS1IPHC502 Reg. No: | | | | | | |

St Aloysius (Deemed to be University)
Mangaluru
Semester I - P.G. Examination - M.Sc. Mathematics
November - 2024

REAL ANALYSIS I

Time: 2 ½ Hours **Max.Marks:60**

Answer any FIVE Full questions from the following: **(12x5=60)**

1. Prove that if $x > 0$ and n is a positive integer, then there exists a unique $y > 0$ such that $y^n = x$. **(12)**

2. a) Let A be a countable set and let B_n be the set of all $n-$ tuples of elements of A. Prove that B_n is countable. **(5)**

b) Let A be the set of all sequences of 0's and 1's. Prove that A is uncountable. **(4)**

c) In a metric space, prove that finite intersection of open sets is open. **(3)**

3. Prove that every k-cell in $\mathbb{R}^k$ is compact. **(12)**

4. a) If $\{s_n\}$ is a monotonically increasing sequence of real numbers then prove that $\{s_n\}$ converges if and only if it is bounded. **(5)**

b) If $|x| < 1$ then show that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$. Also if $|x| \geq 1$ then prove that the series diverges. **(7)**

5 a) If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X, then show that $\{p_n\}$ converges in X. **(5)**

b) Prove that e is irrational. **(5)**

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$. **(2)**

6 a) Prove that a mapping f of a metric space Y is continuous on X if and only if $f^{-1}(C)$ is closed in X for every closed set C in Y. **(7)**

b) If f and g are continuous mapping of X into $\mathbb{R}^k$ then show that $f + g$ and $f.g$ are continuous on X. **(5)**

Contd...2

7 a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous. **(6)**

b) Prove that if f is a continuous mapping of a compact metric space X into $\mathbb{R}^k$ then $f(X)$ is closed and bounded and thus f is bounded. **(6)**

8. a) Suppose f is continuous on $[a, b]$, f' exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f and g is differentiable at the point $f(x)$. If $h(t) = g(f(t)), a \leq t \leq b$, then prove that h is differentiable at x and $h'(x) = g'(f(x))f'(x)$. **(5)**

b) State and prove the generalized mean value theorem. **(7)**

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PS1IPSC500

Reg No :

St Aloysius (Deemed to be University)
Mangaluru

Semester I – P.G. Examination – M.Sc. Mathematics

November - 2024

Graph Theory

Time : 2 ½ Hours | Max. Marks: 60

Answer FIVE FULL questions (12x5=60)

1. a. State and prove Euler's theorem. Hence deduce that in a graph the number of points of odd degree is even. 5

   b. Prove that a graph is bipartite if and only if it does not contain any odd cycle. 7

2. Define the intersection number $\omega(G)$ of a graph G. If G be a connected graph with $p > 3$ points and q lines, then show that $\omega(G) = q$ if and only if G has no triangles. 12

3. a. Prove that the following statements are equivalent for a point v of a connected graph G. 5
   i. v is a cut-point of G.
   ii. There exist points u and w distinct from v such that v is on every $u - w$ path.
   iii. There exists a partition of the set of points $V - \{v\}$ into subsets U and W such that for any point $u \in U$ and $w \in W$, the point v is on every $u - w$ path.

   b. Show that a cubic graph G has a cut point if and only if it has a bridge. 7

4. a. Define the center, radius, diameter, and centroid of a graph with an example. 4

   b. Let G be a connected graph with atleast three points. Prove that the following statements are equivalent: 8
   i) G is a block
   ii) Every two points of G lie on a common cycle.

5. a. Prove that a graph G with atleast $2n$ points is $n-$connected if and only if for any two disjoint sets V_1 and V_2 of n points each, there exist n disjoint paths joining these two set of points. 7

   b. For any graph G, show that $\kappa(G) \leq \lambda(G)$. 5

6. a. Prove that the following statements are equivalent for a connected graph G : 5
   i) G is Eulerian.
   ii) Every point of G has even degree.
   iii) The set of lines of G can be partitioned into cycles.

   b. Prove that for every planar graph G with $p \geq 4$ points has atleast four points of degree not exceeding 5. 7

7. a. Give an example for each of the following: 4
   i) A graph which is both Eulerian and Hamiltonian
   ii) A graph which is Eulerian but not Hamiltonian
   iii) A graph which is Hamiltonian but not Eulerian.

   b. For any graph G with p points, prove that 8
   i. $2p \leq \chi + \bar{\chi} \leq p + 1$
   ii. $p \leq \chi\bar{\chi} \leq (\frac{p+1}{2})^2$

8. a. For any graph G, prove that $\chi(G) \leq 1 + \Delta(G)$, where $\Delta(G)$ is the maximum degree of a point in G. 5

   b. Prove that any planar graph is $5-$colorable. 7

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PSHPSC501 Reg No :

St Aloysius (Deemed to be University), Mangaluru
M.Sc. Mathematics - SEMESTER I PG EXAMINATION
November - 2024
Operations Research

Time : 2 ½ Hours **Max. Marks : 60**

Answer FIVE FULL questions **(12x5=60)**

1. a. Solve using simplex method: 10

Minimize $z = 3x_1 + 5x_2 + 4x_3$
subject to $2x_1 + 3x_2 \leq 8$
$2x_1 + 5x_3 \leq 10$
$3x_1 + 2x_2 + 4x_3 \leq 15$
$x_1, x_2, x_3 \geq 0$

b. Write down the characteristics of canonical form of a linear programming problem. 2

2. a. A firm manufactures two products A and B on which the profit earned per unit are ₹ 3 and ₹ 4 respectively.Each product is processed on two machines M_1 and M_2.Product A requires $1\,minute$ of processing time on M_1 and $2\,minutes$ on M_2.Also B requires $1\,minute$ on M_1 and $1\,minute$ on M_2.The machine M_1 is available for not more than $7\,hours\ 30\,minutes$ while M_2 is available for $10\,hours$ during any working day.Form a linear programming problem and solve it graphically to find out the number of units of A and B to be manufactured to maximize the profit. 6

b. A firm manufactures four different machine parts M_1, M_2, M_3, M_4 made of copper and zinc.The amount of copper and zinc required for each machine part,their exact availability and profit earned from one unit of each machine part are as follows: 6

	$M_1(kg)$	$M_2(kg)$	$M_3(kg)$	$M_4(kg)$	Exact Availability
Copper	5	4	2	1	100
Zinc	2	3	8	1	75
Profit	12	8	14	10	

How many of each part should be manufactured to maximize the profit?

3. a. Write the dual of the following: 4

Maximize $z_y = 3y_1 + y_2 + 2y_3 - y_4$
subject to $2y_1 + y_2 + 6y_3 + y_4 = 1$
$y_1 + y_2 - y_3 + y_4 = 3$
$y_1, y_2 \geq 0, y_3, y_4$ - unrestricted

b. Solve using two phase method : 8

Minimize $z = 3x_1 + 2x_2$
subject to $2x_1 + x_2 \leq 2$
$3x_1 - 4x_2 \geq 12$
$x_1, x_2 \geq 0$

4. Solve the given transportation problems using Vogel's approximation method and verify the optimality of the solution using $MODI$ method. 12

	D_1	D_2	D_3	D_4	Supply
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Demand	5	8	7	14	

5. a. A computer centre has three expert programmers.The centre wants three application programmes to be developed.The head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minutes required by the experts for the application programmes as follows: 6

Programmes/Programmers	A	B	C
1	120	100	80
2	80	90	110
3	110	140	120

Assign the programmers to the programme in such a way that the total computer time is minimum.

Contd...2

b. Write down the methods for finding initial solution using least cost method.Obtain the initial basic feasible solution to the following transporation problem using least cost method and North west corner method. 6

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	5	8
O_3	5	2	2	1	10
Demand	4	6	8	6	

6. a. Explain the Hungarian method for solution of assignment problem. 6

b. Consider the following unbalanced transportation problem. 6

To

From	D_1	D_2	D_3	Supply
O_1	5	1	7	10
O_2	6	4	6	80
O_3	3	2	5	15

The demands for D_1 , D_2 and D_3 are 75 , 20 and 50 respectively.Since the supply is not enough,some of the demands at these destinations may not be satisfied.Suppose there is a penalty for every unsatisfied unit of demand which are 5 , 3 and 2 for destination D_1 , D_2 and D_3 respectively.Find the optimal solution.

7. a. Explain the method of odds to solve a 2×2 game without saddle point.Hence solve: 4

	B_1	B_2
A_1	1	5
A_2	4	2

b. Solve the game whose payoff matrix is given below using rule of dominance: 8

Player A/Player B	1	2	3	4	5
I	2	4	3	8	4
II	5	6	3	7	8
III	6	7	9	8	7
IV	4	2	8	4	3

8. a. Write down the algorithm for solving $m \times 2$ matrix games.Hence solve : 8

	B_1	B_2
A_1	1	2
A_2	5	4
A_3	-7	9
A_4	-4	-3
A_5	2	1

b. Solve the pay off matrix : 4

Player B

Player A	B_1	B_2	B_3	B_4	B_5
A_1	-2	0	0	5	3
A_2	3	2	1	2	2
A_3	-4	-3	0	-2	6
A_4	5	3	-4	2	-6

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